RMO–1996

1. The sides of a triangle are three consecutive integers and its inradius is four units. Determine the circumradius.
2. Find all triples (a, b, c) of positive integers such that
$$(1 + \frac{1}{a})(1 + \frac{1}{b})(1 + \frac{1}{c}) = 3.$$
3. Solve for real number x and y:
$$\begin{aligned} xy^2 &= 15x^2 + 17xy + 15y^2 \\ x^2y &= 20x^2 + 3y^2. \end{aligned}$$
4. Suppose N is an n-digit positive integer such that
   (a) all the n-digits are distinct; and
   (b) the sum of any three consecutive digits is divisible by 5.

   Prove that n is at most 6. Further, show that starting with any digit one can find a six-digit number with these properties.
5. Let ABC be a triangle and h_a the altitude through A. Prove that
$$(b + c)^2 \geq a^2 + 4h_a^2.$$
   (As usual a, b, c denote the sides BC, CA, AB respectively.)
6. Given any positive integer n show that there are two positive rational numbers a and b, $a \neq b$, which are not integers and which are such that $a - b$, $a^2 - b^2$, $a^3 - b^3, \ldots, a^n - b^n$ are all integers.
7. If A is a fifty-element subset of the set $\{1, 2, 3, \ldots, 100\}$ such that no two numbers from A add up to 100 show that A contains a square.