# RMO–1994

1. A leaf is torn from a paperback novel. The sum of the numbers on the remaining pages is 15000. What are the page numbers on the torn leaf.
2. In the triangle $ABC$, the incircle touches the sides $BC$, $CA$ and $AB$ respectively at $D$, $E$ and $F$. If the radius of the incircle is 4 units and if $BD$, $CE$ and $AF$ are consecutive integers, find the sides of the triangle $ABC$.
3. Find all 6-digit natural numbers $a_1a_2a_3a_4a_5a_6$ formed by using the digits 1, 2, 3, 4, 5, 6, once each such that the number $a_1a_2\ldots a_k$ is divisible by $k$, for $1 \leq k \leq 6$.
4. Solve the system of equations for real $x$ and $y$ :

$$\begin{aligned} 5x\left(1+\frac{1}{x^2+y^2}\right) &= 12 \\ 5y\left(1-\frac{1}{x^2+y^2}\right) &= 4. \end{aligned}$$

5. Let $A$ be a set of 16 positive integers with the property that the product of any two distinct numbers of $A$ will not exceed 1994. Show that there are two numbers $a$ and $b$ in $A$ which are not relatively prime.
6. Let $AC$ and $BD$ be two chords of a circle with center $O$ such that they intersect at right angles inside the circle at the point $M$. Suppose $K$ and $L$ are the mid-points of the chord $AB$ and $CD$ respectively. Prove that $OKML$ is a parallelogram.
7. Find the number of all rational numbers $m/n$ such that
   (a) $0 < m/n < 1$
   (b) $m$ and $n$ are relatively prime
   (c) $mn = 25!$
8. If $a$, $b$ and $c$ are positive real numbers such that $a+b+c=1$, prove that

$$(1+a)(1+b)(1+c) \geq 8(1-a)(1-b)(1-c).$$