INMO–2000

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1. The incircle of triangle ABC touches the sides BC, CA and AB in K, L and M respectively. The line through A and parallel to LK meets MK in P and the line through A and parallel to MK meets LK in Q. Show that the line PQ bisects the sides AB and AC of the triangle ABC.

2. Solve for integers x, y, z:
$$x + y = 1 - z, x^3 + y^3 = 1 - z^2.$$

3. If a, b, c, x are real numbers such that $abc \neq 0$ and
$$\frac{xb + (1-x)c}{a} = \frac{xc + (1-x)a}{b} = \frac{xa + (1-x)b}{c}$$
then prove that $a = b = c$.

4. In a convex quadrilateral $PQRS, PQ = RS, (\sqrt{3}+1)QR = SP$ and $\angle RSP - \angle SPQ = 30°$. Prove that
$$\angle PQR - \angle QRS = 90°.$$

5. Let a, b, c be three real numbers such that $1 \geq a \geq b \geq c \geq 0$. Prove that if λ is a root of the cubic equation $x^3 + ax^2 + bx + c = 0$ (real or complex), then $|\lambda| \leq 1$.

6. For any natural numbers $n, (n \geq 3)$, let $f(n)$ denote the number of noncongruent integer-sided triangles with perimeter n (e. g., $f(3) = 1, f(4) = 0, f(7) = 2$). Show that

   (a) $f(1999) > f(1996)$;

   (b) $f(2000) = f(1997)$.