Answers

Theoretical Problem No. 3

Why are stars so large?

1) A first, classic estimate of the temperature at the center of the stars.

1a	We equate the initial kinetic energy of the two protons to the electric potential energy at the distance of closest approach: $2(\frac{1}{2} m_p\, v_{rms}^2) = \frac{q^2}{4\pi\varepsilon_0 d_c}$; and since $\frac{3}{2} k T_c = \frac{1}{2} m_p\, v_{rms}^2$, we obtain $T_c = \frac{q^2}{12\pi\varepsilon_0 d_c k} = 5.5\times 10^9\ \mathrm{K}$	1.5

2) Finding that the previous temperature estimate is wrong.

2a	Since we have that $\frac{\Delta P}{\Delta r} = -\frac{G M_r \rho_r}{r^2}$, making the assumptions given above, we obtain that: $P_c = \frac{G M \rho_c}{R}$. Now, the pressure of an ideal gas is $P_c = \frac{2\rho_c k T_c}{m_p}$, where k is Boltzmann´s constant, T_c is the central temperature of the star, and m_p is the proton mass. The factor of 2 in the previous equation appears because we have two particles (one proton and one electron) per proton mass and that both contribute equally to the pressure. Equating the two previous equations, we finally obtain that: $T_c = \frac{G M m_p}{2 k R}$	0.5

2b	From section (2a) we have that: $\frac{M}{R} = \frac{2 k T_c}{G m_p}$	0.5

2c	From section (2b) we have that, for $T_c = 5.5\times 10^9$ K: $\frac{M}{R} = \frac{2kT_c}{G m_p} = 1.4\times 10^{24}$ kg m^{-1}.	0.5
2d	For the Sun we have that: $\frac{M(Sun)}{R(Sun)} = 2.9\times 10^{21}$ kg m^{-1} , that is, three orders of magnitude smaller.	0.5

3) A quantum mechanical estimate of the temperature at the center of the stars

3a	We have that $\lambda_p = \frac{h}{m_p\, v_{rms}}$, and since $\frac{3}{2} k T_c = \frac{1}{2} m_p\, v_{rms}^2$, and $T_c = \frac{q^2}{12\pi\varepsilon_0\, d_c\, k}$, we obtain: $T_c = \frac{q^4 m_p}{24\pi^2 \varepsilon_0^2\, k\, h^2}$.	1.0
3b	$T_c = \frac{q^4 m_p}{24\pi^2 \varepsilon_0^2\, k\, h^2} = 9.7\times 10^6$ K.	0.5
3c	From section (2b) we have that, for $T_c = 9.7\times 10^6$ K: $\frac{M}{R} = \frac{2kT_c}{G m_p} = 2.4\times 10^{21}$ kg m^{-1} ; while for the Sun we have that: $\frac{M(Sun)}{R(Sun)} = 2.9\times 10^{21}$ kg m^{-1}.	0.5

4) The mass/radius ratio of the stars.

4a	Taking into account that $\frac{M}{R} = \frac{2kT_c}{G m_p}$, and that $T_c = \frac{q^4 m_p}{24\pi^2 \varepsilon_0^2\, k\, h^2}$, we obtain: $\frac{M}{R} = \frac{q^4}{12\pi^2 \varepsilon_0^2\, G\, h^2}$.	0.5

5) The mass and radius of the smallest star.

5a	$n_e = \frac{M}{(4/3)\pi R^3\, m_p}$	0.5

5b	$d_e = n_e^{-1/3} = \left(\frac{M}{(4/3)\pi R^3 m_p} \right)^{-1/3}$	0.5
5c	We assume that $d_e \geq \frac{\lambda_e}{2^{1/2}}$. Since $\lambda_e = \frac{h}{m_e v_{rms}(electron)}$, $\frac{3}{2} k T_c = \frac{1}{2} m_e v_{rms}^2 (electron)$, $T_c = \frac{q^4 m_p}{24 \pi^2 \varepsilon_0^2 k h^2}$, $\frac{M}{R} = \frac{q^4}{12 \pi^2 \varepsilon_0^2 G h^2}$, and $d_e = \left(\frac{M}{(4/3)\pi R^3 m_p} \right)^{-1/3}$, we get that $R \geq \frac{\varepsilon_o^{1/2} h^2}{4^{1/4} q m_e^{3/4} m_p^{5/4} G^{1/2}}$	1.5
5d	$R \geq \frac{\varepsilon_o^{1/2} h^2}{4^{1/4} q m_e^{3/4} m_p^{5/4} G^{1/2}} = 6.9 \times 10^7 \text{ m} = 0.10 \; R(Sun)$	0.5
5e	The mass to radius ratio is: $\frac{M}{R} = \frac{q^4}{12 \pi^2 \varepsilon_0^2 G h^2} = 2.4 \times 10^{21}$ kg m^{-1}, from where we derive that $M \geq 1.7 \times 10^{29}$ kg $= 0.09 \; M(Sun)$	0.5

6) Fusing helium nuclei in older stars.

6a	For helium we have that $\frac{4 q^2}{4 \pi \varepsilon_0 m_{He} v_{rms}^2 (He)} = \frac{h}{2^{1/2} m_{He} v_{rms}(He)}$; from where we get $v_{rms}(He) = \frac{2^{1/2} q^2}{\pi \varepsilon_0 h} = 2.0 \times 10^6$ m s^{-1}. We now use: $T(He) = \frac{v_{rms}^2 (He) m_{He}}{3k} = 6.5 \times 10^8$ K. This value is of the order of magnitude of the estimates of stellar models.	0.5